Homework 2

Show your steps for each problem. Type up answers in LaTeX and turn in by October 21.

1. **Jacobian**. Calculate the Jacobian for the following transformations.
   (a) $Z = X^2$.
   (b) $Z = \exp(X)$.
   (c) $Z = 1/X$.
   (d) $Z = \text{logit}(X)$.

2. Starting from $X \sim \text{Gamma}(a, b)$ density function, use the Jacobian for $Z = 1/X$, and derive the density of Z, which is distributed as an Inverse Gamma random variable.

3. **Poisson**. Cont'd from HW 1, problem 2, same prior, sampling density, and data Y.
   (a) Using the minus 2nd derivative log posterior evaluated at the posterior mode, evaluate the FIP fraction of posterior information coming from the prior.
   (b) Write the posterior mean as a convex combination of the prior mean and the data mean.
   (c) Interpret the prior parameters – what is the prior data mean, and prior sample size in this model?

4. **Power Distribution**, cont'd from HW 1, last problem. A colleague suggests transforming $z_i = -\log y_i$ before analyzing.
   (a) What is the density of $z_i|\theta$? (Give name and give formula for the density.)

5. **Power Distribution**. Continued from Homework 1. Use a $\theta \sim \text{Gamma}(a, b)$ prior.
   (a) Calculate the posterior mean, variance, mode, and the negative 2nd derivative log posterior evaluated at the mode.

(b) Using the 2nd derivative - log posterior evaluated at the posterior mode, evaluate the fraction of posterior information coming from the prior.

(c) (cont'd) Is it preferable to use the formula in the previous item or would it be easier/preferable to use the formula FIP = [1/prior variance]/[1/posterior variance]? One sentence: discuss.

(d) Give two other names for the power distribution.

6. Normal approximations.

   (a) Construct two different (algebraic) normal approximations for the gamma posterior from HW 1 problem 2. (posterior mean-variance; posterior mode and 2nd derivative).

   (b) Construct two data/prior examples. [That is, you pick y_1 (or $y_1, \ldots, y_n$ if you like), and also a and b.]

      i. Make one example where the normal approximation(s) to the gamma posterior are good;

      ii. Make one example where the approximation(s) is (are) not very good. Specify your prior parameters and data for each combo.

   (c) For each data/prior combo, plot the posterior and your two normal approximations on a single graph. [So two plots total, each with three densities on it.]

   (d) Generalize: When will the normal approximation be good and when will it be poor? [1-2 sentences.]