Homework 3

Type up answers in LaTeX and turn in by November 4. Write pseudocode in LaTeX and embed (e.g., R, PYTHON, JULIA) code within the same document.

1. **Normal-normal-normal.** Consider the setup

$$\begin{aligned} \mathbf{y} = y_1, \dots, y_{100} &\overset{iid}{\sim} \text{Normal}(\mu, \sigma^2) \\ \mu &\sim \text{Normal}(\mu_0 = 0, \sigma_0^2 = 100) \\ \sigma^2 &\sim \text{InverseGamma}(\alpha = 2, \beta = 10)\,, \end{aligned}$$

   where the second and third lines denote prior specifications, and μ and σ^2 are assumed to be independent *a priori.*

   After simulating $y_1, \dots, y_{100}$ with $\mu = 50$ and $\sigma^2 = 10$ fixed, approximate the joint posterior distribution $p(\mu, \sigma^2 | y_1, \dots, y_{100})$ using

   (a) a 2D Gaussian approximation centered at the posterior mode (e.g., MAP estimators, $\hat{\mu}$ and $\hat{\sigma}^2$) and with covariance given by

$$-\begin{pmatrix} \frac{\partial^2}{\partial \mu^2} \log p(\mu, \sigma^2|\mathbf{y}) & \frac{\partial^2}{\partial \mu \partial \sigma^2} \log p(\mu, \sigma^2|\mathbf{y}) \\ \frac{\partial^2}{\partial \mu \partial \sigma^2} \log p(\mu, \sigma^2|\mathbf{y}) & \frac{\partial^2}{\partial (\sigma^2)^2} \log p(\mu, \sigma^2|\mathbf{y}) \end{pmatrix}^{-1} \Bigg|_{(\mu,\sigma^2)=(\hat{\mu},\hat{\sigma}^2)},$$

   (b) mean field variational inference with variational distribution

$$q(\mu, \sigma^2 | m, v^2, b) = \text{Normal}(\mu | m, v^2) \times \text{InverseGamma}(\sigma^2 | 1, b),$$

   Create 2D contour plots for each of the 2 approximate posteriors.

2. **2D Clutter problem.** Setting $w = 0.5$ and $N = 200$ implement the clutter problem (as described in class notes). Simulate the data generating mixture distribution with 'true' $\boldsymbol{\theta}$ fixed to $(5, 5)$. Approximate the posterior of the mean parameter $\boldsymbol{\theta}$ using

   (a) assumed density filtering

   (b) and expectation propagation.

   Create 2D contour plots for each of the 2 approximate posteriors.

   Hint: simulating from the data generating mixture distribution will require simulating $w_1, \dots, w_{200} \overset{iid}{\sim} \text{Bernoulli}(1/2)$.